# Mass formulae for spherically symmetric stellar configurations in five diemsional space time

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February 7, 2008

**Abstract**

An expression is derived where the mass is connected to an integral over the pressure of gravitating matter in the frame work of five dimensional (5D) space time.

**Keywords:** String theory, Kaluza-Klein

## 1 Introduction

In view of recent developments in superstring theroy and ten dimensional $N = 1$ Yang-Mills supergravity in its field theory limit, need higher dimensional space time is incerasing. For this reason, in the recent years there has been considerable interest in theories with higher dimensinal space-times in which extra dimesions are contracted to a very small size, apparently beyond our ability for measurment. Marciano (1984) has pointed out that the experimental detections of time variation of fundamental constants should be strong evidence for the existence of extra dimensions. Since the world around us is manifestly 4D in the present era, both in supersymmetry and Kaluza-Klein ( K K ) theories we need to understand the mechanisms of reduction in size of the extra dimensions in order to realise the 4D world. Kaluza-Klein theories are interesting approaches to unify gravity and guage fields, while supersymmetry and supergravity give a natural unification of matter and force. Wesson (1983, 84) and Reddy (1999) have studied several aspects of five dimensional space-time in variable mass theroy and bimetric theory of relativity respectively.

Due to the virial theorem we have the following connection between kinetic and potential energies of an isolated gravitating system

$$2T + V = 0 \tag{1}$$

The potential energy of a static celestial body can be given using Newton's law of gravitation by,

$$V \sim G\frac{M^2}{R_s} \tag{2}$$

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while the kinetic energy can be expressed using the pressure which balances configuration by,

$$T \sim pR_s^3 \tag{3}$$

Here, we denote by $R_s$ the radius of the configuration. So we find a qualitative expression widely used in astrophysics

$$M^2 \sim \frac{1}{G}pR_s^4 \tag{4}$$

Avakian (1990) discussed the exact expression of the connection between the mass, pressure and volume for static spherically symmetric configuration in the framework of four dimensinal space-time. In this paper we derived the mass square of a celestial body which is represented as an integral over the pressure distributions taken over the volume of the body in the frameworks of five dimesional spac-time.

## 1.1 The mass formula for static configurations in five dimesional space-time

The five dimensional line element corresponding to a static spherically symmetric configuration in isotropic coordinates is given as

$$ds^2 = c^2 e^\nu dt^2 - e^\lambda [dr^2 + r^2(d\theta_1^2 + sin^2\theta_1 d\theta_2^2 + sin^2\theta_1 sin^2\theta_2 d\theta_3^2)] \tag{5}$$

where $\nu$ and $\lambda$ are functions of $r$-only. The five velocity has vanishing spatial components $u^1 = u^2 = u^3 = u^4 = 0$ and $u^0 u_0 = 1$ so we have the following nonvanishing components of the energy momentum tensor,

$$T_0^0 = \rho, \quad T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p \tag{6}$$

where $p$ denotes the pressure and $r$ is the matter density of the configuration. The Einstein field equations in five dimensions are

$$R_i^j = -\frac{8\pi G}{c^4}[T_i^j - \frac{1}{3} \ T] \tag{7}$$

From (5) and (6), equation (7) can be expressed as,

$$R_0^0 = -\frac{8\pi G}{c^4}[\frac{2}{3}\rho c^2 + \frac{4}{3}p] \tag{8}$$

$$R_1^1 = R_2^2 = R_3^3 = R_4^4 = -\frac{8\pi G}{c^4}[-\frac{1}{3}\rho c^2 + \frac{1}{3}p] \tag{9}$$

where

$$R_0^0 = -\frac{1}{2}e^{-\nu}[\nu'' + \lambda'\nu' + \frac{1}{2}\nu'^2 + 3\frac{\nu'}{r}] \tag{10}$$

$$R_1^1 = -\frac{1}{2}e^{-\lambda}[3\lambda'' + 3\frac{\lambda'}{r} - \frac{1}{2}\nu'\lambda' + \nu'' + 3\frac{1}{2}\nu'] \tag{11}$$

$$R_2^2 = R_3^3 = R_4^4 = -\frac{1}{2}e^{-\lambda}[\lambda'' + 5\frac{\lambda'}{r} + \lambda'^2 + \frac{1}{2}\nu'\lambda' + \frac{\nu}{r}] \tag{12}$$

and prime denotes derivative with respect to $r$. Again from (8) and (9)

$$R_0^0 + 2R_4^4 = -\frac{16\pi Gp}{c^4} \tag{13}$$

From (10) and (12), equation (13) can be written as

$$\frac{16\pi G p}{c^4}e^\lambda = \lambda'' + \frac{\nu''}{2} + \lambda'^2 + \frac{\nu'^2}{4} + \nu'\lambda' + 5\frac{\nu'}{r} \tag{14}$$

Multiply both sides by $r^3 e^{(\lambda+\frac{\nu}{2})}$

$$\frac{16\pi G p}{c^4}r^3 e^{(2\lambda+\frac{\nu}{2})} = [r^3 e^{(\lambda+\frac{\nu}{2})}(\lambda' + \frac{\nu'}{2} + \frac{1}{r})]' - 2e^{(\lambda+\frac{\nu}{2})} \tag{15}$$

Multiply both sides by $r^2$

$$\frac{16\pi G p}{c^4}r^5 e^{(2\lambda+\frac{\nu}{2})} = [r^5 e^{(\lambda+\frac{\nu}{2})}(\lambda' + \frac{\nu'}{2}]' \tag{16}$$

After integrating (16) for $r$ from zero to infinity taking into account that $p(r) = 0$ for $r \geq r_s$, where $r_s$ is the radius of configuration we get,

$$\frac{16\pi G p}{c^4}\int_0^{r_s} p\, r^5 e^{(2\lambda+\frac{\nu}{2})} dr = r^5 e^{(\lambda+\frac{\nu}{2})}(\lambda' + \frac{\nu'}{2}) \tag{17}$$

The five dimensional exterior solution of Einsteins equations in isotropic coordinates are given by,

$$e^\lambda = (1 + \frac{r_g}{4r^2})^2, \quad e^\nu = \frac{(1 - \frac{r_g}{4r^2})^2}{(1 + \frac{r_g}{4r^2})^2} \tag{18}$$

where $r_g = \frac{2GM}{c^2}$ is the gravitational radius of the body and M denotes mass of the body.

Using (18), equation (17) reduces to

$$\frac{16\pi G p}{c^4}\int_0^{r_s} p\, r^5 e^{(2\lambda+\frac{\nu}{2})} dr = \frac{r_g^2}{4} \tag{19}$$

using $r_g = \frac{2GM}{c^2}$, equation (19) is expressed as

$$M^2 = \frac{16\pi}{G}\int_0^{r_s} p\, r^5 e^{(2\lambda+\frac{\nu}{2})} dr \tag{20}$$

## 1.2 The mass formula for static configurations in Kaluza-Klein space-time

Consider the line element in the form,

$$ds^2 = c^2 e^\nu dt^2 - e^\lambda[dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2] - e^\mu dy^2 \tag{21}$$

where $\nu$ & $\lambda$ are functions of $r$ only and $y$ is a Kaluza-Klein parameter.

The nonvanishing Ricci tensors for the metric (21) are

$$R_0^0 = -e^{-\lambda}[\frac{\nu''}{2} + \frac{\nu'^2}{2} + \frac{\lambda'\nu'}{4} + \frac{\nu'\mu'}{4} + \frac{\nu'}{4}] \tag{22}$$

$$R_1^1 = -e^{-\lambda}[\frac{\nu''}{2} + \lambda'' + \frac{\mu''}{2} - \frac{\lambda'\nu'}{4} - \frac{\lambda'\mu'}{4} + \frac{\lambda'}{r} + \frac{\nu'^2}{4} + \frac{\mu'^2}{4}] \tag{23}$$

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$$R_2^2 = R_3^3 = -e^{-\lambda}\left[\frac{\lambda''}{2} + \frac{3\lambda'}{2r} + \frac{\nu'}{2r} + \frac{\mu'}{2r} + \frac{\lambda'\nu'}{4} + \frac{\lambda'\mu'}{4} + \frac{\lambda'^2}{4}\right] \tag{24}$$

$$R_4^4 = -e^{-\lambda}\left[\frac{\mu''}{2} + \frac{\mu'^2}{4} + \frac{\mu'\nu'}{4} + \frac{\mu'\lambda'}{4} + \frac{\mu'}{4}\right] \tag{25}$$

From equations (6) and (7), we have

$$R_0^0 + 2R_2^2 = -\frac{16\pi G}{c^4}p \tag{26}$$

using (22), (24) and (25), equation (26) can be expressed as,

$$\frac{1}{2}[\nu'' + \lambda'' + \mu''] + \frac{1}{4}[\nu'^2 + \lambda'^2 + \mu'^2 + \lambda'\nu' + \lambda'\mu' + +\mu'\nu'] + \frac{3}{2r}[\nu' + \mu' + \lambda'] = -\frac{16\pi G}{c^4}\,p\,e^\lambda \tag{27}$$

**For a particular case $\mu = 0$ :**

Equation (27) reduces to,

$$\frac{1}{2}[\nu'' + \lambda''] + \frac{1}{4}[\nu'^2 + \lambda'^2 + \lambda'\nu'] + \frac{3}{2r}[\nu' + \lambda'] = -\frac{16\pi G}{c^4}\,p\,e^\lambda \tag{28}$$

multiply both sides by $r^2 e^{(\lambda + \frac{\nu}{2})}$

$$\frac{16\pi G}{c^4}pr^2 e^{(3\lambda + \frac{\nu}{2})} = [r^2 e^{(3\lambda + \frac{\nu}{2})}(\frac{\nu' + \lambda'}{2} + \frac{1}{r})]' - e^{(\frac{\lambda + \nu}{2})} \tag{29}$$

multiply both sieds by $r$

$$\frac{16\pi G}{c^4}pr^2 e^{(\frac{3\lambda + \nu}{2})} = [r^3(e^{(\frac{\lambda + \nu}{2})})']' \tag{30}$$

Integrating from $r = 0$ to infinity taking into account that $p(r) = 0$ for $r \geq r_s$, where $r^s$ is the radius of configuration,

$$\frac{16\pi G}{c^4}\int_0^{r_s} p\,r^3 e^{(\frac{3\lambda + \nu}{2})}dr = r^3(e^{(\frac{\lambda + \nu}{2})})' \tag{31}$$

From the five dimensional Schwarzschilds solution in isotropic coordinates (Wessons (1999)) .

$$e^\lambda = (1 + \frac{r_g}{4r})^4, \qquad e^\nu = \frac{(1 - \frac{r_g}{4r^2})^2}{(1 + \frac{r_g}{4r^2})^2} \tag{32}$$

where $r_g = \frac{2GM}{c^2}$ is the gravitational radious of the body.

Using (32), equation (31) reduces to

$$\frac{16\pi G}{c^4}\int_0^{r_s} p\,r^3 e^{(\frac{3\lambda + \nu}{2})}dr = \frac{r_g^2}{8} \tag{33}$$

using $r_g = \frac{2GM}{c^2}$, equation (1.33) can be expressed as

$$M^2 = \frac{32\pi}{G}\int_0^{r_s} p\,r^3 e^{(\frac{3\lambda + \nu}{2})}dr \tag{34}$$

The above expressions of the mass in Kaluza-Klein theory is similar to the expression of the mass obtained earlier by Beylar-et-al (1995) in four dimensional space-time.

## 1.3 Conclusion:

In the present work, we have derived the new formulae for the mass of spherically symmetric stellar configurations in the frameworks of five dimensional space-time. We observed that the mass square of static spherically symmetric celestial body is connected to the pressure distribution in the framework of five dimesional space-time. It is also observbed that in Kaluza-Klein space time the expressions of the mass are similar as that of expression of the mass obtained by Beylar-et-al (1995) in four dimension. This new mass formulae is of importance especially in numerical calculations of the masses of celestial bodies. .

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Submitted: October 18, 2002